QUADRATIC SEQUENCES (2)

Learning Outcomes and Assessment Standards

Learning Outcome 1: Number and Number relationships
Assessment Standards
Investigate number patterns (including but not limited to those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore quadratic) and hence:
(a) Make conjectures and generalization.
(b) Provide explanations and justifications and attempt to prove conjectures.

Overview

In this lesson you will:

- Revise second order difference patterns..
- Study recursive number patterns.

Lesson

Revision of quadratic number patterns

Quadratic number patterns have the general term $T_n = an^2 + bn + c$.

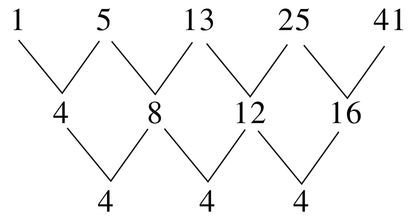
$a + b + c =$ first term

$3a + b =$ first term of first difference row

$2a =$ second difference

Example

Determine the general term of the number pattern 1; 5; 13; 25; 41; …………



It is clearly a quadratic number pattern because it has a constant second difference.

You can now proceed as follows:

$2a = 4$	$3a + b = 4$	$a + b + c = 1$
$\therefore a = 2$	$\therefore 3(2) + b = 4$	$\therefore 2 - 2 + c = 1$
	$\therefore b = -2$	$\therefore c = 1$

Therefore the general term is $T_n = 2n^2 - 2n + 1$

Recursive linear number patterns

Consider the number pattern: 3; 5; 7; 9; 11; ………….

$T_1 = 3$

$T_2 = 3 + 2 = T_1 + 2$

$T_3 = 5 + 2 = T_2 + 2$

$T_4 = 7 + 2 = T_3 + 2$

$T_5 = 9 + 2 = T_4 + 2$

$T_6 = 11 + 2 = T_5 + 2$

$\therefore T_{n+1} = T_n + 2$

This formula represents the recursive form of the number pattern.

Further examples

(a) Express in recursive form:

4; 10; 16; 22; 28; 34; ………..

(b) Determine the number pattern if:

$T_1 = 2$ and $T_n = T_{n-1} + 5$

(a) 4; 10; 16; 22; 28; 34; ………..

$T_1 = 4$

$T_2 = 4 + 6 = T_1 + 6$

$T_3 = 10 + 6 = T_2 + 6$

$T_4 = 16 + 6 = T_3 + 6$

$\therefore T_n = T_{n-1} + 6$

(b) $T_1 = 2$

$T_2 = T_1 + 5 = 2 + 5 = 7$

$T_3 = T_2 + 5 = 7 + 5 = 12$

$T_4 = T_3 + 5 = 12 + 5 = 17$

Therefore the number pattern is:

2; 7; 12; 17; 22; ……………………..

Recursive quadratic number patterns

Example

Express the following number patterns in recursive form:

(a) 2; 5; 10; 17; 26; ………..

(b) 9; 16; 28; 45; 67; ………

(a) $T_1 = 2$

$\therefore T_2 = 2 + 3 = T_1 + 3$

$\therefore T_3 = 5 + 5 = T_2 + 5$

$\therefore T_4 = 10 + 7 = T_3 + 7$

$\therefore T_5 = 17 + 9 = T_4 + 9$

$\therefore T_n = T_{n-1} + ??????$

Now consider the pattern 2; 3; 5; 7; 9; …..

The first term 2 is the odd man out in the pattern because 3; 5; 7; 9; … is linear with a constant difference of 2 between the terms. With the first term, 2, you have to add 1 to get to the second term, 3.

What you can do is work out the general term for the pattern 3; 5; 7; 9; … and then subtract the odd man out.

For the linear pattern 3; 5; 7; 9; ……

$T_n = 3 + (n - 1)(2)$

$\therefore T_n = 3 + 2n - 2$

$\therefore T_n = 2n + 1$

Now let's remove the odd man out by subtracting 1 from *n*:

$T_n = 2(n - 1) + 1$

$\therefore T_n = 2n - 2 + 1$

$\therefore T_n = 2n - 1$

If we now add in $(2n - 1)$ to the recursive formula, we will get:

$T_n = T_{n-1} + (2n - 1)$

We can now verify this recursive formula as follows:

$T_1 = 2$

$T_2 = T_1 + (2(2) - 1) = 2 + 3 = 5$

$T_3 = T_2 + (2(3) - 1) = 5 + 5 = 10$

$T_4 = T_3 + (2(4) - 1) = 10 + 7 = 17$

Clearly, the recursive formula generates the terms of the number pattern.

(b) 9; 16; 28; 45; 67; ………

$T_1 = 9$

$\therefore T_2 = 9 + 7 = T_1 + 7$

$\therefore T_3 = 16 + 12 = T_2 + 12$

$\therefore T_4 = 28 + 17 = T_3 + 17$

$\therefore T_5 = 45 + 22 = T_4 + 22$

$\therefore T_n = T_{n-1} + ??????$

Now consider the pattern 9; 7; 12; 17; 22; …..

The first term 9 is the odd man out in the pattern because is linear with a constant difference of 5 between the terms. With the first term, 9, you have to subtract 2 to get to the second term, 7.

What you can do is work out the general term for the pattern 7; 12; 17; 22; …..

and then subtract the odd man out.

For the linear pattern 7; 12; 17; 22; …..

$T_n = 7 + (n - 1)(5)$

$\therefore T_n = 7 + 5n - 5$

$\therefore T_n = 5n + 2$

Now let's remove the odd man out by subtracting 1 from *n*:

$T_n = 5(n - 1) + 2$

$\therefore T_n = 5n - 5 + 2$

$\therefore T_n = 5n - 3$

If we now add in $(5n - 3)$ to the recursive formula, we will get:

$T_n = T_{n-1} + (5n - 3)$

We can now verify this recursive formula as follows:

$T_1 = 9$

$T_2 = T_1 + (5(2) - 3) = 9 + 7 = 16$

$T_3 = T_2 + (5(3) - 3) = 16 + 12 = 28$

$T_4 = T_3 + (5(4) - 3) = 28 + 17 = 45$

Clearly, the recursive formula generates the terms of the number pattern.

Activity 1

1. Write each of the following number patterns in recursive form.
    (a) 4; 7; 10; 13; ……………………………..
    (b) 0; 4; 8; 12; ………………………………
    (c) 10; 6; 2; ………………………………..
2. Determine the number pattern in each of the following cases:
    (a) $T_1 = 3$ and $T_n = T_{n-1} + 3$
    (b) $T_1 = 2$ and $T_n = 3T_{n-1}$

Activity 2

Write each of the following number patterns in recursive form.

(a) 7; 24; 51; 88; ……………………………..

(b) 3; 12; 27; 48; ……………………………..

(c) 6; 17; 34; 57; ……………………………..